



μετά την κρούση

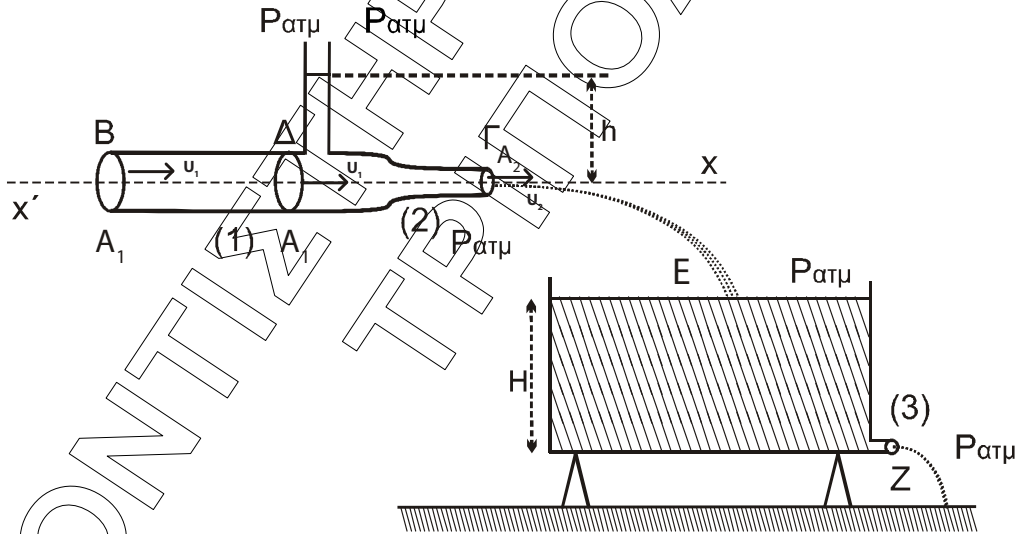
$$f_2 = \frac{v_{\eta\zeta}}{v_{\eta\zeta} + v_k} f_s = \frac{v_{\eta\zeta}}{v_{\eta\zeta} + \frac{v_s}{2}} f_s \quad (3)$$

$$\text{Από } \frac{(2)}{(3)} \Rightarrow \frac{f_1}{f_2} = \frac{\frac{v_{\eta\zeta}}{v_{\eta\zeta} + v_s} f_s}{\frac{v_{\eta\zeta}}{v_{\eta\zeta} + v_k} f_s} = \frac{v_{\eta\zeta} + v_k}{v_{\eta\zeta} + v_s} =$$

$$= \frac{v_{\eta\zeta} + \frac{v_{\eta\zeta}}{40}}{v_{\eta\zeta} + \frac{v_{\eta\zeta}}{20}} = \frac{41}{42}$$

Άρα σωστό το (ii)

B2.



Εξ. συνέχ. Από Δ → Γ

$$\Pi_1 = \Pi_2 \Rightarrow A_1 v_1 = A_2 v_2 \Rightarrow 2A_2 \cdot v_1 = A_2 v_2 \Rightarrow \underline{v_2 = 2v_1} \quad (1)$$

Bernoulli: Δ → Γ

$$\left. \begin{aligned} P_{\Delta} + \frac{1}{2}\rho v_1^2 &= P_2 + \frac{1}{2}\rho v_2^2 \\ \text{Στον κατακόρυφο σωλήνα} &\Rightarrow P_{\text{atm}} + \rho gh + \frac{1}{2}\rho v_1^2 = P_{\text{atm}} + \frac{1}{2}\rho v_2^2 \Rightarrow \\ P_{\Delta} &= P_{\text{atm}} + \rho gh \end{aligned} \right\}$$

$$\Rightarrow gh + \frac{1}{2}v_1^2 = \frac{1}{2}v_2^2 \Rightarrow gh + \frac{1}{2}\frac{v_2^2}{4} = \frac{1}{2}v_2^2 \Rightarrow$$

$$\Rightarrow \frac{3}{8}v_2^2 = gh \Rightarrow v = \sqrt{\frac{8}{3}gh} \quad (2)$$

$$\left. \begin{aligned} \text{Στο δοχείο η επιφάνεια σταθερή σε ύψος (H)} \\ \text{άρα:} \end{aligned} \right\} \Rightarrow A_2 v_2 = A_3 v_3 \Rightarrow$$

$$\Rightarrow A_2 v_2 = \frac{A_2}{2} v_3 \Rightarrow v_3 = 2v_2$$

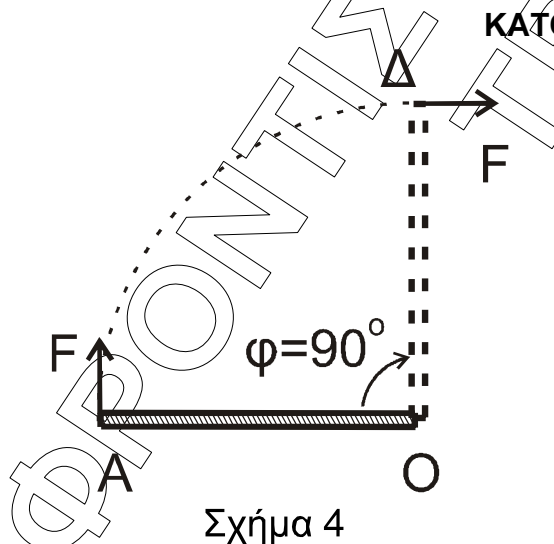
Bernoulli: E \rightarrow Z

$$P_{\text{atm}} + \rho gH + 0 = P_{\text{atm}} + \frac{1}{2}\rho v_3^2 + 0 \Rightarrow gH = \frac{1}{2}4v_2^2 \Rightarrow$$

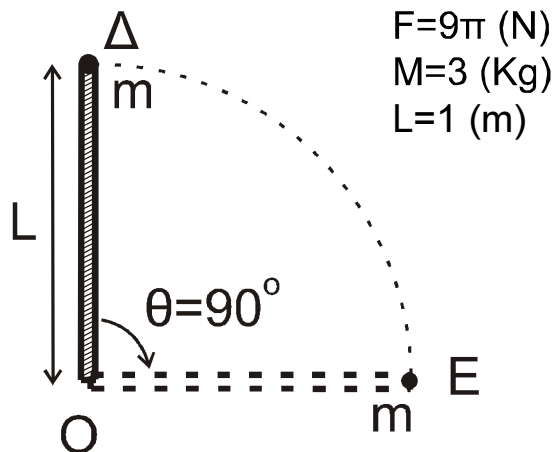
$$\Rightarrow gH = 2v_2^2 \Rightarrow v_2 = \sqrt{g\frac{H}{2}} \text{ από την (2)} \Rightarrow \sqrt{\frac{8}{3}gh} = \sqrt{g\frac{H}{2}} \Rightarrow \frac{h}{H} = \frac{3}{16}$$

Σωστό (iii)

B3



Σχήμα 4



Σχήμα 5

Για την κίνηση A \rightarrow Δ από το Θ.Μ.Κ.Ε. ισχύει:

$$\Delta K = \Sigma W \Rightarrow \frac{1}{2} \cdot I_0 \cdot \omega_\Delta^2 = (F \cdot L) \frac{\pi}{2} \Rightarrow \frac{1}{2} \cdot \frac{1}{3} \cdot M \cdot L^2 \cdot \omega_\Delta^2 = F \cdot L \cdot \frac{\pi}{2} \Rightarrow$$

$$\Rightarrow \frac{1}{2} \cdot \frac{1}{3} \cdot 3 \cdot 1 \cdot \omega_\Delta^2 = 9\pi \cdot 1 \cdot \frac{\pi}{2} \Rightarrow \omega_\Delta = 3\pi \text{ rad/s}$$

Από Α.Δ.Σ. στην κρούση στο (Δ) ισχύει:

$$\vec{L}_{\text{πριν}} = \vec{L}_{\text{μετά}} \Rightarrow I_0 \cdot \omega_\Delta = I'_0 \cdot \omega'_\Delta \Rightarrow \omega'_\Delta = \frac{I_0 \cdot \omega_\Delta}{I'_0} \quad (1)$$

$$\text{Όμως } I'_0 = \frac{1}{3} M \cdot L^2 + mL^2 = \frac{3 \cdot 1^2}{3} + 1 \cdot 1^2 = 2 \text{ Kgm}^2 \quad (2)$$

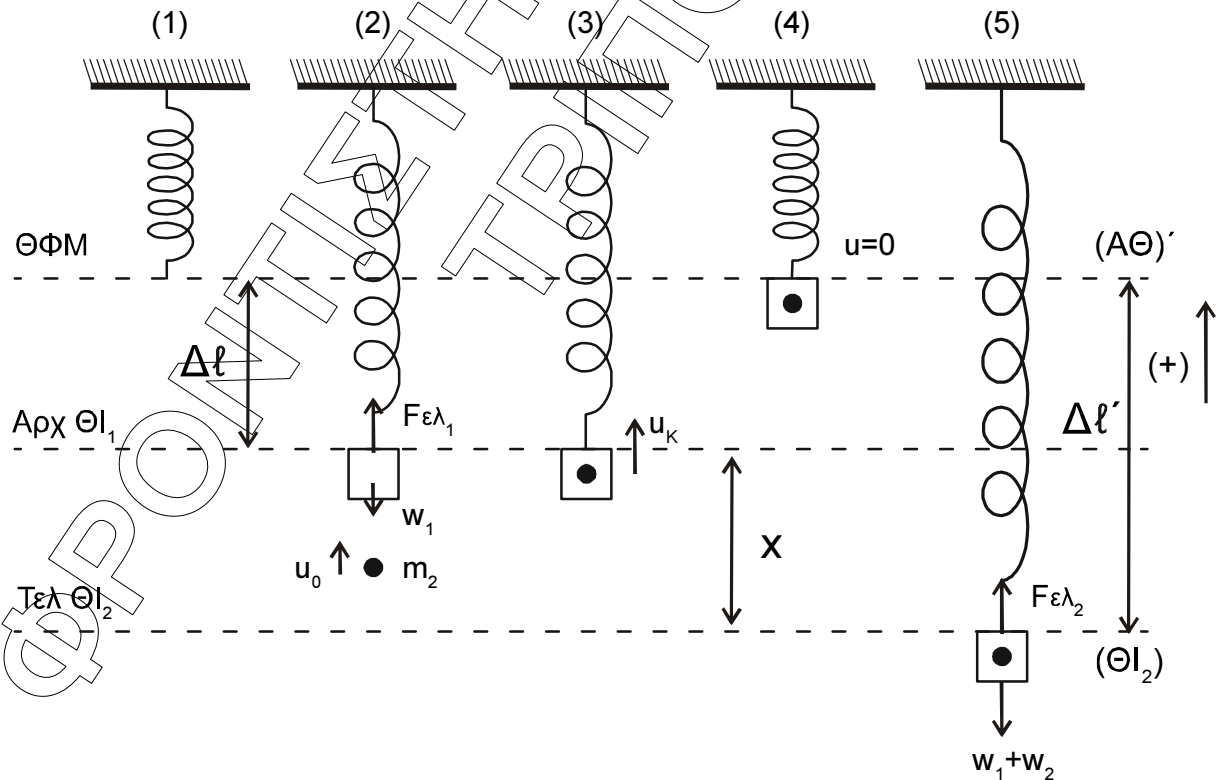
$$\text{Από (1) και (2) έχουμε: } \omega'_\Delta = \frac{\frac{ML^2}{3} \cdot \omega_\Delta}{2} = \frac{3 \cdot 1 \cdot 3\pi}{3 \cdot 2} = \frac{3\pi}{2} \text{ rad/s}$$

Για τον χρόνο $t_{\Delta \rightarrow E} = t$ έχουμε

$$\Delta\Theta = \omega'_\Delta \cdot t \Rightarrow \frac{\pi}{2} = \frac{3\pi}{2} \cdot t \Rightarrow t = \frac{1}{3} \text{ (s)}$$

Άρα σωστό είναι το (ii).

ΘΕΜΑ Γ



Γ1. Για την αρχική Θ.Ι.1 σχήμα (2) ισχύει:

$$\Sigma F_y = 0 \Rightarrow W_1 = F_{ελ.1} \Rightarrow m_1 \cdot g = K \cdot \Delta l \Rightarrow K = \frac{10}{0,05} = 200 \text{ N/m}$$

Για την τελική Θ.Ι.2, σχήμα (5)

$$\Sigma F_y = 0 \Rightarrow W_1 + W_2 = F_{ελ.2} \Rightarrow (m_1 + m_2) \cdot g = K \cdot \Delta l' \Rightarrow 20 = 200 \cdot \Delta l' \Rightarrow \Delta l' = 0,1 \text{ m}$$

Άρα το πλάτος ΑΘ'-ΘΙ.2: $\Delta l' = A = 0,1 \text{ m}$

Γ2. $x = \Delta l' - \Delta l = 0,05 \text{ m}$

ΑΔΕΤ:

$$K + U = E_T \Rightarrow \frac{1}{2} \cdot (m_1 + m_2) \cdot v_K^2 + \frac{1}{2} \cdot k \cdot x^2 = \frac{1}{2} \cdot k \cdot A^2 \Rightarrow 2 \cdot v_K^2 + 200 \cdot 0,05^2 = 200 \cdot 0,1^2 \Rightarrow$$

$$\Rightarrow v_K^2 = 1 - 0,25 \Rightarrow v_K = \sqrt{0,75} = \frac{\sqrt{3}}{2} \text{ m/s}$$

$$\text{Από: } \overrightarrow{P}_{πριν} = \overrightarrow{P}_{μετά} \Rightarrow m \cdot v_o = 2 \cdot m \cdot v_K \Rightarrow v_o = \frac{\sqrt{3}}{2} \cdot 2 = \sqrt{3} \text{ m/s}$$

$$\text{Άρα } K = \frac{1}{2} \cdot m \cdot v_o^2 = \frac{1}{2} \cdot 1 \cdot \sqrt{3}^2 = \frac{3}{2} = 1,5 \text{ J}$$

Γ3.

$$\Delta \overrightarrow{P}_2 = \overrightarrow{P}'_2 - \overrightarrow{P}_2 = \Delta P_2 = m_2 v_k - m_2 v_o \Rightarrow$$

$$\Delta P_2 = \frac{\sqrt{3}}{2} - \sqrt{3} \Rightarrow \Delta P_2 = -\frac{\sqrt{3}}{2} \text{ kg m/s} \Rightarrow$$

$$\Rightarrow |\Delta P_2| = \frac{\sqrt{3}}{2} \text{ kg m/s}$$

Με κατεύθυνση προς τα κάτω, προς τα αρνητικά.

Γ4. Για $t = 0, x = 0,05 \text{ m}, A = 0,1 \text{ m}, v > 0$

$$x = A \eta \mu(\omega t + \varphi_0) \Rightarrow 0,05 = 0,1 \eta \mu \varphi_0 \Rightarrow \eta \mu \varphi_0 = \frac{1}{2} = \eta \mu \frac{\pi}{6}$$

$$\text{άρα } \varphi_0 = 2k\pi + \frac{\pi}{6}$$

$$0 \leq \varphi_0 < 2\pi \Bigg\} \kappa=0 \Rightarrow$$

$$\text{ή } \varphi_0 = 2k\pi + \frac{5\pi}{6}$$

$$\left. \begin{array}{l} \varphi_0 = \frac{\pi}{6} \text{ με } v > 0 \text{ δεκτή} \\ \varphi_0 = \frac{5\pi}{6} \text{ } v < 0 \end{array} \right\} \text{άρα } \varphi_0 = \frac{\pi}{6}$$

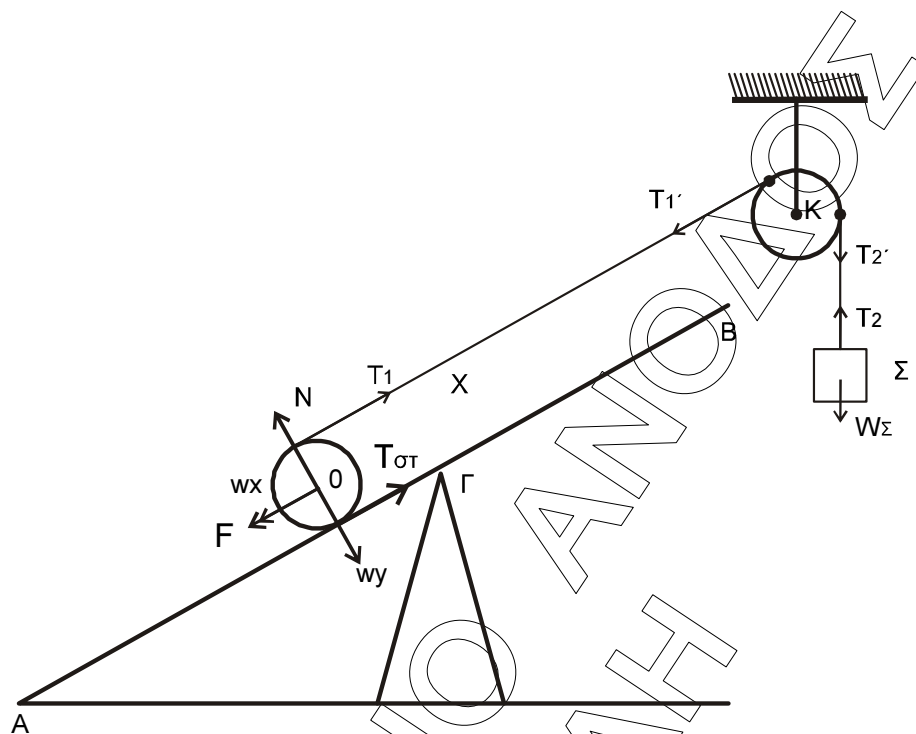
$$w = \sqrt{\frac{k}{m_1 + m_2}} = \sqrt{100} = 10 \text{ rad/s}$$

Άρα η απομάκρυνση είναι

$$x = 0,1 \eta \mu \left(10t + \frac{\pi}{6} \right) \text{ (SI)}$$

ΘΕΜΑ Δ

Δ1.



Σώμα (Σ):

$$\left. \begin{aligned} \Sigma F_y = 0 &\Rightarrow T_2 = W_\Sigma \\ T_2' = T_2 &\end{aligned} \right\} T_2' = W_\Sigma \Rightarrow T_2' = 20 \text{ N}$$

Στην τροχαλία:

$$\Sigma \tau_{(K)} = 0 \Rightarrow T_1' R_T = T_2' R_T \Rightarrow T_1' = T_2' \Rightarrow T_1' = 20 \text{ N}$$

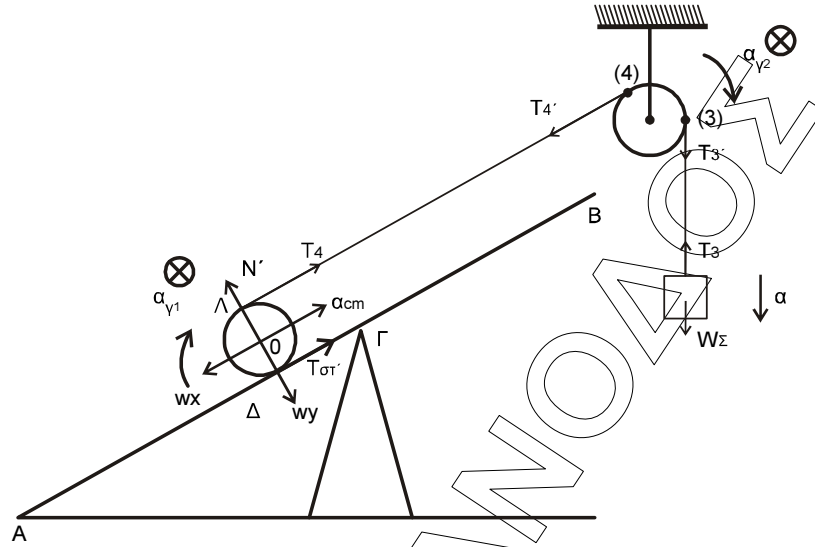
$$T_1 = T_1' = 20 \text{ N}$$

Στον κύλινδρο:

$$\Sigma \tau_{(O)} = 0 \Rightarrow T_1' R_K = T_{\sigma\tau} R_K \Rightarrow T_{\sigma\tau} = 20 \text{ N}$$

$$\Sigma F_x = 0 \Rightarrow F + W_x = T_1 + T_{\sigma\tau} \Rightarrow F = 40 - 10 \Rightarrow \underline{F = 30 \text{ N}}$$

Δ2.



$T_3 = T'_3$, $T_4 = T'_4$ αβαρή νήματα

Για τον κύλινδρο

μεταφ: $\Sigma F_x = m_k \cdot a_{cm} \Rightarrow T_4 + T'_{\sigma\tau} - W_x = m_k \cdot a_{cm}$ (1)

στροφ: $\Sigma \tau_{(0)} = I_0 \cdot \alpha_{\gamma_1} \Rightarrow T_4 \cdot R_k - T'_{\sigma\tau} \cdot R_k = m_k \frac{R_k^2}{2} \alpha_{\gamma_1}$ (2)

Στην τροχαλία:

στροφ: $\Sigma \tau_k = I_k \cdot \alpha_{\gamma_2} \Rightarrow T'_3 \cdot R_T - T'_4 \cdot R_T = m_T \frac{R_T^2}{2} \alpha_{\gamma_2}$ (3)

Στο σώμα:

μεταφ: $\Sigma F_y = m_\Sigma \cdot a \Rightarrow W_\Sigma - T_3 = m_\Sigma \cdot a$ (4)

η ταχύτητα του σώματος $v_\Sigma = v_3$ άρα $\alpha_3 = \alpha \Rightarrow \alpha = \alpha_{\gamma_2} R_T$ (5)

η ταχύτητα $v_\Lambda = v_4$ άρα $\alpha_\Lambda = \alpha_4 = \alpha_{\gamma_2} \cdot R_T$

όμως $v_\Lambda = 2v_{cm}$ άρα $\alpha_\Lambda = 2a_{cm}$

$v_\Delta = 0$ άρα $v_{cm} = \omega \cdot R_k$ άρα $\alpha_{cm} = a_{\gamma_1} R_k$ (7)

$2a_{cm} = \alpha$ (6)

Λύνοντας:

Από (2),(7) $\Rightarrow (T'_4 - T'_{\sigma\tau}) R_k = \frac{m_k R_k^2}{2} \frac{\alpha_{cm}}{R_k} \Rightarrow T'_4 - T'_{\sigma\tau} = \frac{m_k}{2} \alpha_{cm} \stackrel{(m_k=2)}{\Rightarrow} T'_4 - T'_{\sigma\tau} = \alpha_{cm}$ (8)

Από (3),(5) $\Rightarrow (T_3 - T_4) R_T = \frac{m_T R_T^2}{2} \frac{\alpha}{R_T} \Rightarrow T_3 - T_4 = \frac{m_T}{2} \alpha \stackrel{(m_T=2)}{\Rightarrow} T_3 - T_4 = \alpha$ (9)

Από (8),(6) $\Rightarrow T_4 - T'_{\sigma\tau} = \frac{\alpha}{2}$ (10)

(4) $20 - T_3 = 2\alpha$
 (9) $T_3 - T_4 = \alpha$ } $20 - T_4 = 3\alpha$ (11)

$$\text{Από (1)(6)} \Rightarrow T_4 + T_{\sigma\tau} - m_K g \eta \mu \varphi = m_K \cdot a_{cm}$$

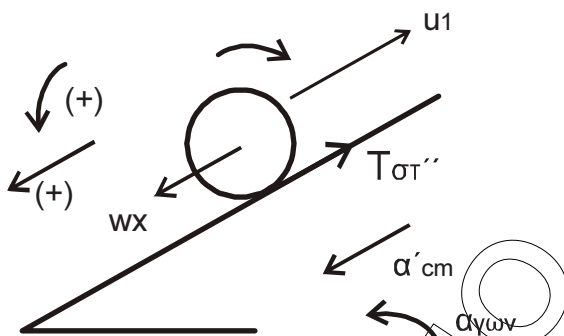
$$T_4 + T_{\sigma\tau} - 2 \cdot 10 \cdot \frac{1}{2} = 2 \frac{a}{2} \Rightarrow T_4 - T_{\sigma\tau} = 10 + a \quad (12)$$

$$\text{Από (10), (12)} \Rightarrow 2T_4 = 10 + \frac{3a}{2} \Rightarrow$$

$$2(20 - 3a) = 10 + \frac{3a}{2} \Rightarrow 40 - 6a = 10 + \frac{3a}{2} \Rightarrow a = 4 \frac{\text{m}}{\text{s}^2}$$

$$(6) \quad a_{cm} = \frac{a}{2} = 2 \frac{\text{m}}{\text{s}^2}.$$

Δ3.



$$v_1 = \alpha_{cm} \cdot t = 2 \cdot 0,5 = 1 \text{ m/s}$$

$$\Sigma F_x = M \cdot \alpha_{cm}$$

$$W_x - T_{\sigma\tau}'' = M_K \cdot \alpha_{cm}'$$

$$M_K \cdot g \cdot \eta \mu \varphi - T_{\sigma\tau}'' = M_K \cdot \alpha_{cm}' \quad (1)$$

$$\Sigma \tau = I_K \cdot \alpha'_{\gamma\omega\nu} \Rightarrow T_{\sigma\tau}'' \cdot R_K = \frac{M_K R_K^2}{2} \alpha'_{\gamma\omega\nu} \quad (2)$$

$$\text{κύλιση } \alpha'_{cm} = \alpha'_{\gamma\omega\nu} \cdot R_K \quad (3)$$

$$\text{Από (2) και (3)} \quad T_{\sigma\tau}'' \cdot R_K = M_K \frac{R_K^2}{2} \cdot \frac{\alpha'_{cm}}{R_K} \Rightarrow T_{\sigma\tau}'' = \frac{M_K \cdot \alpha'_{cm}}{2} = \frac{2 \cdot \alpha'_{cm}}{2} \Rightarrow$$

$$\Rightarrow T_{\sigma\tau}'' = \alpha'_{cm} \quad (4)$$

$$(1), (4) \quad 2 \cdot 10 \cdot \frac{1}{2} - \alpha'_{cm} = 2 \alpha'_{cm} \Rightarrow 10 = 3 \alpha'_{cm} \Rightarrow$$

$$\Rightarrow \alpha'_{cm} = \frac{10}{3} \text{ m/s}^2$$

$$v = v_1 - \alpha'_{cm} \cdot \Delta t \Rightarrow 0 = 1 - \frac{10}{3} \cdot \Delta t \Rightarrow \Delta t = \frac{3}{10} = 0,3 \text{ s}$$

$$t_{\text{STOP}} = 0,5 + \Delta t = 0,8 \text{ s}$$

